Name and surname

Osnove matematične analize: 2. računski izpit

2. febuar 2021

Student ID

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Time limit is 60 minutes. You may use 2 A4-sized sheets of paper with formulas. The use of electronic devices (calculator, phone) is prohibited. Justify all your answers!
Write each problem onto a separate sheet of paper. If you are writing onto a blank piece of paper rather than the problem sheet, please sign your name at the top of every page, write the problem number at the top as well and scan the problems in the correct order. Thanks!

Question 1 (35 marks)

a) (10 marks) Find the complex numbers u, v so that the equation

$$(z^2 + u)(z^3 + v) = z^5 + 4z^3 + 8iz^2 + 32i$$

holds for all complex numbers z.

b) (15 marks) Find all the solutions to the equation

$$z^5 + 4z^3 + 8iz^2 + 32i = 0$$

If you were unable to solve **a)** part of the exercise then solve the equation $z^3 - 8i = 0$ (but in this case the exercise is worth only 10 marks).

c) (10 marks) Find all the solutions to the equation

$$|z|^3z^2 + 4|z|^3 - 8iz^2 - 32i = 0$$

Hint: Compare this equation to the expression from **a)**.

Question 2 (30 marks)

Let the sequence (a_n) be given by the recursive formula

$$a_{n+1} = \sqrt{3 + 2a_n}$$

and the initial term $a_1 = \sqrt{3}$.

a) (20 marks) Graphically represent the terms of the given sequence by graphing the functions $f(x) = x$ and $g(x) = \sqrt{3 + 2x}$ on the same coordinate system and precisely sketch the intersection of the two curves. Is the sequence bounded? Is it monotonous? Prove your answers with induction.

b) (10 marks) Is the given sequence conergent? If yes, calculate its limit $\lim_{n\to\infty} a_n$. Does your answer to the last question change if the initial term is $a_1 = 4$? Briefly explain why.

Question 3 (35 marks)

Define a function $f\colon \mathbb{R}^2 \to \mathbb{R}$,

$$f(x, y) = (x + 2)y - (y - 2)x^2.$$

a) (10 marks) Find all stationary points of f.

b) (15 marks) Determine the equation of the level curve through the stationary point with the largest x-coordinate. Plot the level curve.

c) (5 marks) Does this level curve contain self-intersections? Is this stationary point a saddle?

d) (5 marks) Compute the Hesse matrix of the map f at this stationary point.