Oma 3. izpit, rešitve

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① a) $z^3+8i=0$

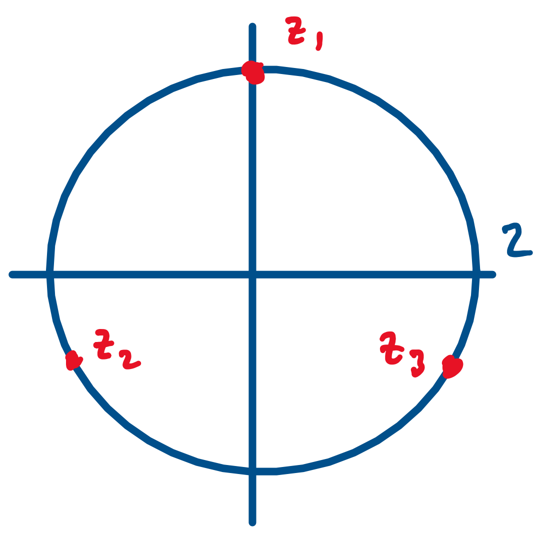
$z^3 = -8i = 2^3 e^{i\left(\frac{3\pi}{2}+2k\pi\right)}$, $k\in\mathbb{Z}$.

$()^{1/3} \Rightarrow z = 2e^{i\frac{\pi}{2}} e^{i\frac{2k\pi}{3}}$

Torej: $z_1 = 2e^{i\frac{\pi}{2}} = 2i$

$z_2 = 2e^{i\frac{\pi}{2}} e^{i\frac{2\pi}{3}} = 2e^{i\frac{7\pi}{6}}$

$z_3 = 2e^{i\frac{11\pi}{6}}$



b) $z^3\bar z - z^4 + iz^3 + 8i\bar z - 8iz - 8 = 0$.

LHS $= (z^3+8i)(az+b\bar z+c) =$

$= bz^3\bar z + az^4 + cz^3 + 8ib\bar z + 8iaz + 8ic$

$\Rightarrow b=1,\ a=-1,\ c=i$.

Rešujemo torej: $(z^3+8i)(-z+\bar z+i)=0$.

- če je $z^3+8i=0$, so z_1, z_2 in z_3 iz a)-dela rešitve dane enačbe.
- če je $-z+\bar z+i=0$, zapišimo $z=x+iy$ in tedaj dobimo $-(x+iy)+x-iy+i=0$ oz. $-2iy+i=0$.

To pomeni, da je $y=\frac{1}{2}$, x pa je poljuben.

$\Rightarrow z = x+\frac{1}{2}i$ za $x\in\mathbb{R}$ je rešitev.

Vse rešitve so: $\{z_1,z_2,z_3\}\cup\{x+\frac{1}{2}i;\ x\in\mathbb{R}\}$.

② a) $\lim_{x\to0} x^2\log(x^2) = ?$

$? = \lim_{x\to0}\frac{\log(x^2)}{\frac{1}{x^2}} \overset{L'H}{=} \lim_{x\to0}\frac{\frac{1}{x^2}2x}{-2x^{-3}} = -\lim_{x\to0}x^2 = 0$.

b) $\int x^2\log(x^2)\,dx = ?$

$u=\log(x^2)$ $\quad du = \frac{1}{x^2}2x\,dx = \frac{2}{x}dx$

$dv = x^2dx$ $\quad v=\frac{x^3}{3}$

$? = \frac{x^3}{3}\log(x^2) - \int\frac{2}{3}x^2dx$

$= \frac{x^3}{3}\log(x^2) - \frac{2}{9}x^3 + C$

c) $\int_{-1}^{1} x^2\log(x^2)\,dx = ?$

$? \overset{b)}{=} \frac{x^3}{3}\log(x^2) - \frac{2}{9}x^3\Big|_{x=-1}^{1}$; $\log(1)=0$

$= -\frac{2}{9}x^3\Big|_{x=-1}^{1} = -\frac{4}{9}$

③ a) $f(x,y)=x^3+y^3-xy$

Stacionarne tč.:

$f_x = 3x^2-y = 0$

$f_y = 3y^2-x = 0$

$\Rightarrow x = 3y^2 \Rightarrow x = 27x^4$

$x(1-27x^3)=0$

$\boxed{x=0,\ y=0}$ ali $\boxed{x=\frac{1}{3},\ y=3\cdot\left(\frac{1}{3}\right)^2=\frac{1}{3}}$

Klasifikacija:

$H(x,y) = \begin{bmatrix}6x & -1\\ -1 & 6y\end{bmatrix} \Rightarrow \det(H(x,y)) = 36xy-1$

- $(0,0)$: $\det(H(0,0)) = -1 \Rightarrow$ sedlo.
- $(\frac{1}{3},\frac{1}{3})$: $\det(H(\frac{1}{3},\frac{1}{3})) = \frac{36}{9}-1 = 3 > 0$

in pa $f_{xx}(\frac{1}{3},\frac{1}{3}) = \frac{6}{3} = 2 > 0$

$\Rightarrow$ lokalni minimum.

b) $F(x,y,\lambda) = y - \lambda(x^3+y^3-1)$

$F_x = -\lambda 3x^2 = 0$

$F_y = 1-\lambda 3y^2 = 0$

$F_\lambda = 0$ ... vez

↳ če je $\lambda=0$, $F_y=0$ ne velja.

Zato je $x=0$.

Iz vezi pa tedaj sledi $y^3=1 \Rightarrow y=1$.

Torej $(0,1)$ je vezani ekstrem.