Computational Topology

Exam 2, July 10, 2015

You have 90 minutes to solve the problems.

You are allowed to use your notes. You are not allowed to use calculators, ipads, phones, neighbors or other devices. Any form of communication is forbidden and will result in a negative grade in all past and current work in this class.

1. A simplicial complex K with vertices 0,1,2,3,4,5,6 contains the following triangles

$$\{\{0,1,2\},\{0,2,3\},\{0,3,4\},\{0,4,5\},\{0,5,1\},\{1,2,6\},\{2,3,6\},\{3,4,6\},\{4,5,6\},\{5,1,6\}\}.$$

   (a) Show that K represents the triangulation of a surface.

   (b) Compute its Euler characteristic.

   (c) If you know that the surface is orientable (you do not have to check this), can you recognize which surface it is? Write down its Betti numbers.

2. Define a distance measure between two words as the minimal number of positions in the words with different letters. A blanks (empty space) is considered as an additonal letter in the alphabet. For example, the distance between FLIP and CHIP is 2, and the distance between NEXT and TEXTS is also 2.

   Using this distance measure on the set

$$S = \{SONCE, SENCE, SRE\check{C}A, VRAG, SRENJ\}$$

   (a) build the filtration of Vietoris-Rips complexes

$$VR_1(S) < VR_2(S) < \cdots < VR_5(S),$$

   (b) construct the corresponding barcodes and persistence diagram.

3. For the simplicial complex on the whiteboard:

   (a) write down the boundary matrices of the chain complex in dimensions 0 and 1,

   (b) define a discrete Morse function on the complex (with integer values between 0 and 10),

   (c) construct the corresponding discrete vector field,

   (d) write down the boundary matrices of the Morse chain complex in dimensions 0 and 1,

   (e) write down the ranks of the groups of cycles Z_0 and Z_1, the groups of boundaries B_0 and B_1, and the Betti numbers in dimension 0 and 1.