Computational Topology

Exam 1, June 17, 2015

You have 90 minutes to solve the problems.

You are allowed to use your notes. You are not allowed to use calculators, ipads, phones or neighbors. Any form of communication is forbidden and will result in a negative grade in all past and current work in this class.

1. Given the sample of points $S = \{x_1 = (0,0), x_2 = (1,1), x_3 = (2,3), x_4 = (-1,2), x_5 = (3,-1), x_6 = (4,2), x_7 = (5,0)\}$,
   (a) construct a triangulation on S by sweeping a vertical line from left to right,
   (b) list all triangles in the star of $\mathrm{St}(x_2)$.
   (c) Is the obtained triangulation Delaunay? Why?
   (d) How many edge flips are necessary to produce a Delaunay triangulation?

2. The Klein bottle K is obtained by gluing opposite sides of a square in the following way: the bottom and top sides are oriented in the same direction and glued accordingly, while the left and right sides are oriented in the opposite directions and glued accordingly.
   (a) Triangulate the Klein bottle by cutting up the square into 18 triangles (as in the case of the torus), number the vertices from 1 to 18.
   (b) Write down a 1-cycle in K that is a boundary.
   (c) Write down a 1-cycle in K that is not a boundary.
   (d) Is there any 2-cycle in K? Is it a boundary?
   (e) Compute the Euler characteristic $\chi(K)$.
   (f) What are the Betti numbers of K?

3. You are given a list of pairs of the form (σ, f), where σ is a simplex of a simplicial complex K and f is the value of a function defined on K:

$$\{(v_0,0),(v_1,2),(v_2,2),(v_3,0),(v_4,5),(\langle v_0,v_1\rangle,4),(\langle v_0,v_2\rangle,1),(\langle v_0,v_3\rangle,4),$$
$$(\langle v_1,v_2\rangle,3),(\langle v_1,v_3\rangle,1),(\langle v_0,v_1,v_2\rangle,4),(\langle v_3,v_4\rangle,4)\}.$$

   (a) Is f a discrete Morse function? Why?
   (b) If it is, draw the corresponding discrete vector field and list the critical simplices.
   (c) Write down the Morse chain complex and the matrices corresponding to the boundary operators ∂_1 and ∂_2.
   (d) What are the Betti numbers of K?

4. Write down the filtration by sublevel complexes for the function f on the simplicial complex from the previous problem and draw the persistence diagrams in dimension 0 and 1.