Mathematical modelling

22. 8. 2018

1. Determine the values of a and b so that the graph of the function
$$f(x) = a\cos x + b\sin x$$
will be as close as possible to the points $(0, 0)$, $(\pi/2, 1)$ in $(\pi/4, 0)$:
   (a) write down the system of equations a and b given by the point coordinates,
   (b) find the Moore–Penrose inverse A^+ of the matrix A of the system,
   (c) find the values for the best fit using A^+.

2. Given the vector-valued function $\mathbf{f}(u, v) = \begin{bmatrix} u \\ v \\ u^2 + v^2 \end{bmatrix}$
   (a) write down its Jacobian matrix,
   (b) write down its linear approximation at the point $u = 1, v = 1$,
   (c) sketch the parametric surface in $\mathbb{R}^3$ given by $\mathbf{f}(u, v)$. Find the equation of its tangent plane at the point $u = 1, v = 1$.

3. Given the parametric curve $\mathbf{p}(t) = \begin{bmatrix} t + \frac{1}{t} \\ 2\log t \end{bmatrix}$
   (a) write the equation of the line tangent to the curve at $t = 2$,
   (b) compute the arc length between the points $t = 1$ and $t = 3$.

4. A can of beer cooled to 5°C is taken out of the fridge and put into a room with temperature 20°C. After 25 minutes the beer has 10°C.
   (a) Write down the differential equation describing how the temperature of the can changes with time. Use Newton's law which says that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature.
   (b) What will the temperature of the beer be after 20 minutes?
   (c) When will it reach the ideal temperature of 15°C?