Mathematical modelling, Exam 2

5. 7. 2019

1. The system of equations $2x - y + z = 3$ and $-x + 2y - z = 1$ can be expressed in the form $Ax = b$.
   (a) Find the Moore-Penrose inverse of A, $A^\dagger$.
   (b) Describe the property uniquely characterizing the point $A^\dagger b$ with respect to the system.
   (c) Construct any single matrix, which has the following matrices as their generalized inverses: $\begin{bmatrix} 3 & 2 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 3 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$.

2. Given the parametric curve $\gamma(t) = [2\cos(t), 2\sin(t), -t]^\mathsf{T}$:
   (a) Sketch/describe γ.
   (b) Parameterize γ with a natural parameter.
   (c) Find the center and the radius of the osculating circle to γ at the point $(2, 0, 0)$.
   (d) Find the length of γ between points $(2, 0, 0)$ and $(2, 0, 2\pi)$.

3. Find the solution y of the differential equation $x^2y' + xy + 3 = 0$ with the initial condition $y(1) = 1$.

4. Solve the following system of differential equations:

$$x'(t) = -2x(t) + 5y(t),$$
$$y'(t) = x(t) + 2y(t),$$

   with the initial conditions $x(0) = y(0) = 1$.