

Mathematical modelling

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1. We are given four points: $(0, 1)$, $(-1, 0)$, $(1, 2)$, $(2, 3)$. We would like to fit a function of the form $ax^2 + bx$ to these points.
 - (a) Write down the matrix A of the corresponding system of linear equations.
 - (b) Find the Moore-Penrose inverse A^+ .
 - (c) Find the function of the above form that fits the points best according to the least squares criterion.
 - (d) Find one more generalized inverse of A .
2. Given the parametric curve $\gamma(t) = (t^3 - t + 1, t^2)$:
 - (a) Find selfintersections of γ .
 - (b) Find the angle at which γ intersects itself in the selfintersections.
 - (c) Find the point at which γ reaches its lowest level (smallest y coordinate).
3. Solve the differential equation $xy' = y + 2x^3$ with the initial condition $y(2) = 3$.
4. Solve the differential equation $y'' + y' - 6y = 36x$. with the initial condition $y(0) = y'(0) = 1$.