

# Izpit iz Matematičnega modeliranja

21. 8. 2020

This is an open book exam. You are allowed to use your notes, books and any other literature. You are NOT allowed to use any communication device. You have 105 minutes to solve the problems.

1. Let

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ -1 & 1 \end{bmatrix} \quad \text{in} \quad b = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$$

- Find the Moore–Penrose inverse  $A^+$  of the matrix  $A$ .
- Does the system  $Ax = b$  have a solution? Why?
- If the system is solvable, find the solution closest to the origin, if not, find the vector  $x^+$  such that the error  $\|Ax^+ - b\|$  is the smallest possible.
- Find the Moore–Penrose inverse  $(A^+)^+$  of  $A^+$ .

2. Given the parametric curve  $\vec{r}(t) = (x(t), y(t)) = (t^3 - 4t, t^2 - 4)$ .

- find all points where it intersects the coordinate axes,
- find the tangent to the curve at  $t = 1$ ,
- find the points on the curve where the tangent is horizontal or vertical,
- if there is a selfintersection, find it and compute the area inside the loop formed by the curve,
- sketch the curve.

3. Find the general solution to the differential equation

$$y' = 2x(1 + y^2)$$

and the particular solution that satisfies  $y(1) = 0$ .

4. For the system of nonlinear differential equations

$$\dot{x} = xy + 1, \quad \dot{y} = x + xy$$

- find its stationary point,
- classify it as a saddle, source, sink or center,
- sketch the phase portrait of the system around the stationary point.