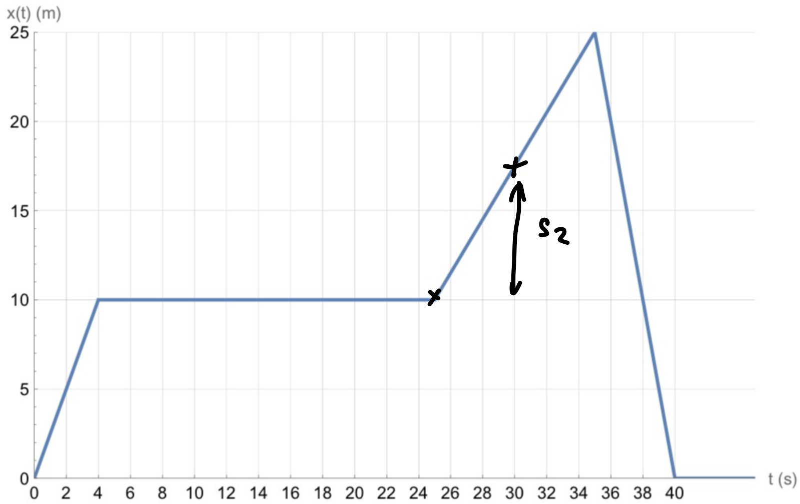
1. Gimnastičarka Simona je na olimpijskih igrah svojo rutino opravljala po diagonali igrišča, ki meri $d = 25$ m. Tekom rutine se je njena oddaljenost od roba spreminjala kot je prikazano na grafu.

   a) Kolikšno pot je opravila v prvih 30 s svoje rutine? Kolikšna pa je celotna pot, ki jo je opravila v celotni rutini?

   b) Kolikšno hitrost je imela ob časih $t_1 = 3$ s in $t_2 = 38$ s?



a) pot v prvih 30s.

$s_1 = 10m$ (odčitano) 3

$s_2 = ?$

$$\left.\begin{array}{l} x(25s) = 10m \\ x(35s) = 25m \end{array}\right\} \quad v = \frac{25m - 10m}{10s} = 1.5m/s$$ 2

za s_2 štejo tudi logično razmišljanje: 30 je pol med 25 in 30 potem je pol od 15 = 7.5 (5)

$$x(30s) = x(25s) + v \cdot t$$
$$= 10m + 1.5m/s \cdot 5s$$
$$= 17.5m$$

$\Rightarrow s_2 = 7.5m$ 5

$\Rightarrow$ opravi 17.5m poti

pot v celi rutini: začne v izhodišču, gre na drugi rob, ki je oddaljen $d = 25m$ in se vrne:

$$s = 2 \cdot d = \underline{\underline{50m}}$$ 5

①

b) hitrost v $t_1 = 3s$. enakomerno gibanje med 0s in 4s:

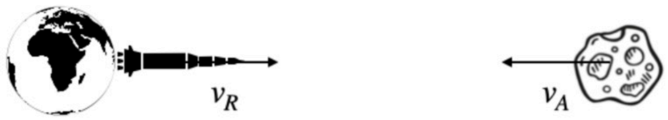
$$v(3s) = \frac{\Delta x}{\Delta t} = \frac{10m - 0m}{4s - 0s} = 2.5 m/s \quad 5$$

hitrost v $t_2 = 38s$. enakomerno gibanje med 35s in 40s:

$$v(38s) = \frac{\Delta x}{\Delta t} = \frac{0m - 25m}{40s - 35s} = -\frac{25m}{5s} = -5 m/s \quad 5$$

(3 brez predznaka)

2. Znanstveniki so na razdalji opazili asteroid z maso $m_A = 10^7$ kg, ki potuje naravnost proti zemlji s hitrostjo $v_A = 10$ km/s. Da bi preprečili, da asteroid pade na Zemljo, so predlagali, da proti njemu izstrelijo balistične rakete (brez eksploziva), ki bodo z njim prožno trčile na razdalji $d = 10^5$ km. Koliko takih raket bi bilo potrebno izstreliti naravnost v asteroid, da odbijemo asteroid. Upoštevaj, da vsaka balistična raketa tehta $m_R = 12000$ kg in da jih izstrelimo s hitrostjo, ki ravno zadošča, da odbiti asteroid pobegne Zemljini gravitaciji. Rakete po izstrelitvi ne kurijo goriva. Masa Zemlje $m_Z = 6 \times 10^{24}$ kg, radij Zemlje pa $R_Z = 6400$ km.



prožni trk → ohranitev gibalnih količin in energij

ENERGIJE

začetek: $\frac{m_R v_R^2}{2} - \frac{G m_R m_Z}{R_Z} + \frac{m_A v_A^2}{2} - \frac{G m_A m_Z}{R_Z + d}$

konec: $\frac{m_R v_R'^2}{2} - \frac{G m_R m_Z}{R_Z + d} + \frac{m_A v_A'^2}{2} - \frac{G m_A m_Z}{R_Z + d}$

5 (energije)

GIBALNE KOL:

$$m_R v_R - m_A v_A = m_R v_R' + m_A v_A'$$

5 (gibalne)

g: 3 za zač ali kon

RAZMISLEK:

$v_A' = v_{ubežna}(d)$

$v_R' = 0$ besedilo

$\Rightarrow \quad m_R v_R - m_A v_A = m_A v_A'$

raketa moramo izračunati ravno hitrost, da da asteroidu ubežno hitrost

=> poenostavitev enačb

$$Z: \frac{m_R v_R^2}{2} - \frac{G m_R m_Z}{R_Z} + \frac{m_A v_A^2}{2} - \frac{G m_A m_Z}{R_Z+d}$$

$$K: -\frac{G m_R m_Z}{R_Z+d} + \frac{m_A v_A'^2}{2} - \frac{G m_A m_Z}{R_Z+d}$$

ohranitev energij

$$\frac{m_R v_R^2}{2} - \frac{G m_R m_Z}{R_Z} + \frac{m_A v_A^2}{2} - \cancel{\frac{G m_A m_Z}{R_Z+d}} =$$

$$= -\frac{G m_R m_Z}{R_Z+d} + \frac{m_A v_A'^2}{2} - \cancel{\frac{G m_A m_Z}{R_Z+d}}$$

$$0 = \frac{m_R v_R^2}{2} - \frac{G m_R m_Z}{R_Z} + \frac{m_A v_A^2}{2} + \frac{G m_R m_Z}{R_Z+d} - \frac{m_A v_A'^2}{2}$$

pa še

$$m_R v_R - m_A v_A = m_A v_A'$$

$$v_R = \frac{m_A}{m_R}\left(v_A' + v_A\right)$$

$$0 = \frac{m_R}{2}\frac{m_A^2}{m_R^2}\left(v_A' + v_A\right)^2 - \frac{G m_R m_Z}{R_Z} + \frac{m_A v_A^2}{2} + \frac{G m_R m_Z}{R_Z+d} - \frac{m_A v_A'^2}{2} \quad 5$$

$$0 = \frac{m_A^2}{2m_R}\left(v_A' + v_A\right)^2 - m_R\left(\frac{G m_Z}{R_Z} - \frac{G m_Z}{R_Z+d}\right) + \frac{m_A}{2}\left(v_A^2 - v_A'^2\right)$$

$$0 = \frac{c}{m_R} + a m_R + b \quad / \cdot m_R$$

$$0 = c + a m_R^2 + b m_R \quad 3 \quad \text{kvadratna enačba}$$

$$m_R = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = -m_e\left(\frac{Gm_z}{R_z} - \frac{Gm_z}{R_z+d}\right) = -5.377 \cdot 10^{12}$$

$$b = \frac{m_A}{2}\left(v_A^2 - v_A'\right)^2 = 4.62 \cdot 10^{14}$$

$$c = \frac{m_A^2}{2}\left(v_A' + v_A\right)^2 = 8.119 \cdot 10^{21}$$

ubežna hitrost na d:

$$\frac{1}{2}\cancel{m_A} v_A'^2 = \frac{G m_z \cancel{m_A}}{R_z+d}$$

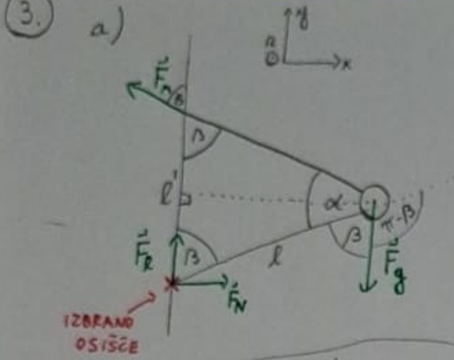
$$v_A' = \sqrt{\frac{2Gm_z}{R_z+d}} = 2742\ m/s \qquad 5$$

ko rešimo kvadratno enačbo dobimo:

$$m_e = 37707\ kg$$

$\Rightarrow$ rabimo torej 4 rakete $(4 \cdot 12000\ kg > 37000\ kg)$ 2

3. a)



$$\sum \vec{M}_i = 0 \rightarrow \underline{M_a:} \quad 3$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$\ell' F_n \sin\beta - \ell F_g \sin(\pi-\beta) = 0$$

$$\ell' F_n \cancel{\sin\beta} - \ell mg \cancel{\sin\beta} = 0 \qquad \sin\beta \neq 0$$

$$2 \quad 2\cancel{\ell} \sin\frac{\alpha}{2} F_n = \cancel{\ell} mg$$

$$\boxed{F_n = \frac{mg}{2\sin\frac{\alpha}{2}} = 769\ \mathrm{N}} \ (4)$$

$$\frac{\ell'}{2\ell} = \sin\frac{\alpha}{2}$$

$$2 \quad \ell' = 2\ell \sin\frac{\alpha}{2}$$

$$\alpha + 2\beta = \pi$$

$$\beta = \frac{\pi-\alpha}{2}$$

$$\sin\beta = \sin\left(\frac{\pi}{2}-\frac{\alpha}{2}\right) = \cos\frac{\alpha}{2}$$

$$\cos\beta = \cos\left(\frac{\pi}{2}-\frac{\alpha}{2}\right) = \sin\frac{\alpha}{2}$$

b) $f = \dfrac{F_n}{mg} = \dfrac{1}{2\sin\frac{\alpha}{2}}$, $f_{max} = 1{,}5$

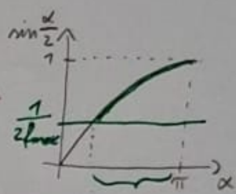
POGOJ, DA SE USPE DRŽATI:

$$f \leq f_{max} \quad 2$$

$$\frac{1}{2\sin\frac{\alpha}{2}} \leq f_{max}$$

$$\sin\frac{\alpha}{2} \geq \frac{1}{2f_{max}}$$

$$\frac{\alpha}{2} \geq \arcsin\frac{1}{2f_{max}}$$



$$\boxed{\alpha \geq 2\arcsin\frac{1}{2f_{max}} = 38{,}9^\circ} \quad 3$$

c) POGOJ, DA JI NE ZDRSI:

$$F_\ell \leq k F_N \quad 2$$

$$\sum \vec{F}_i = 0 \rightarrow \underline{x:} \quad -F_n \sin\beta + F_N = 0$$

$$F_N = \frac{mg}{2\sin\frac{\alpha}{2}} \sin\beta = \frac{mg}{2\tan\frac{\alpha}{2}} \quad 3 \quad 3$$

$$\underline{y:} \quad F_n\cos\beta + F_\ell - mg = 0$$

$$\frac{mg}{2\cancel{\sin\frac{\alpha}{2}}}\cancel{\sin\frac{\alpha}{2}} + F_\ell = mg$$

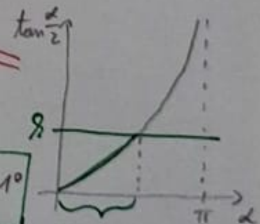
$$F_\ell = \frac{mg}{2} \quad 3$$

$$F_\ell \leq k F_N$$

$$\frac{\cancel{mg}}{2} \leq k \frac{\cancel{mg}}{2\tan\frac{\alpha}{2}}$$

$$\tan\frac{\alpha}{2} \leq k$$

$$\frac{\alpha}{2} \leq \arctan k$$



$$\boxed{\alpha \leq 2\arctan k = 53{,}1^\circ} \quad 2$$

JANJA SE LAHKO ZADRŽI SAMO PRI TEH α!

SKUPNI POGOJ

$$2\arcsin\frac{1}{2f_{max}} \leq \alpha \leq 2\arctan k$$

$$38{,}9^\circ \leq \alpha \leq 53{,}1^\circ$$

4

$\rho = 1{,}8 \cdot 10^{-2} \frac{\Omega mm^2}{m}$ $\qquad S = a^2 \qquad P = 0{,}2\, mm^2$

$B(t) = B_0 e^{-\alpha t} \cos(\omega t + \frac{\pi}{3})$ $\qquad R = \frac{\rho \cdot l}{P}$ 3 $= \frac{4\rho a}{P} = \cancel{2{,}2\ldots\Omega}$

$0{,}0072\,\Omega$ 2

$B_0 = 2T$

$\alpha = 2s^{-1}$

$\omega = 500^{-1}$

$U_i = -\frac{d\Phi_m}{dt}$ 5 $= -S \frac{dB(t)}{dt} = -S B_0 \frac{d}{dt}\left(e^{-\alpha t} \cos(\omega t + \frac{\pi}{3})\right)$

$U_i = -S B_0 e^{-\alpha t}\left(-\alpha \cos(\omega t + \frac{\pi}{3}) - \omega \sin(\omega t + \frac{\pi}{3})\right)$ 10

(op: 5 za odvod produkta)

$t_1 = 0{,}45 s$

$I = \frac{U_i}{R}$ 3 $= -2{,}26 A$ 2

5) Najprej izračunajmo nadomestno upornost R_N

$$R_{123} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = 0{,}545\,\Omega$$

$$R_{1235} = R_{123} + R_5 = 5{,}545\,\Omega$$

$$R_N = \left(\frac{1}{R_4} + \frac{1}{R_{1235}}\right)^{-1} = 2{,}32\,\Omega \qquad 5$$

$\sum U = 0$ 2. kirchhoffov zakon

$$\frac{e}{C_1} + R_N \cdot I = 0 \quad \Big| \text{ upoštevamo } I = \frac{de}{dt}$$

5

$$-\frac{e}{C_1} = R_N \frac{de}{dt} \quad \Big| \int$$

$$-\frac{1}{R_N C_1}\int dt = \int \frac{de}{e}$$

$$-\frac{t}{R_N C_1} = \ln \frac{e(t)}{e_0} \Rightarrow e(t) = e_0 e^{-t/R_N C_1}$$

5 (kul tudi za tok)

$$I(t) = \frac{de}{dt} = -\frac{e_0}{R_N C_1} e^{-\frac{t}{R_N C_1}}$$

$e(t) = 0{,}4\,As \quad e_0 = 0{,}97\,As \quad t = ?$

a) $$t = -R_N C_1 \ln\left(\frac{e(t)}{e_0}\right) = 14{,}39\,ms$$ 3 2

$$I(t) = -\frac{e(t)}{C_1 R_N} = -\frac{e_0}{R_N C_1} e^{-\frac{t}{R_N C_1}}$$

ali-ali

$I(0{,}05\,s) = 2{,}75\,A$

Kolikšen tok je skozi R_1?

$$U_4 = U_{1235}$$

$$I_4 R_4 = I_{1235} R_{1235}$$

$$\Rightarrow I_{1235} = \frac{I}{1 + \frac{R_{1235}}{R_4}} = 1{,}15\,A = I_5 = I_{123}$$

$$U_{123} = R_{123} \cdot I_{123} = R_1 I_1$$

b) $$I_1 = \frac{R_{123} I_{123}}{R_1} = 0{,}63\,A$$ 5