1. The system of equations

$$x - y + z - w = 1$$
$$x + y - z - w = 3$$

determines a two-dimensional plane in \mathbb{R}^4 . Let T(0,-1,-1,2). Our objective is to find the point on the plane which is closest to T.

- (a) Write the matrix *A* and the right-hand side **b** of the system above.
- (b) Evaluate A^+ . (This is simple since A has full rank).
- (c) Show that $P = I A^+A$ is an orthogonal projection, meaning that $P^2 = P$ and $P^T = P$. Onto which subspace does it project?
- (d) Express and compute the solution using A^+ .
- (e) Write the function pT = projekcija(A, b, T), which returns the projection of the point T on to the hyperplane defined by the system Ax = b.
- 2. **Eigenvalues of (symmetric) matrices.** Suppose that $A \in \mathbb{R}^{n \times n}$ is a matrix with eigenvalues $\lambda_1, \ldots, \lambda_n$ such that $|\lambda_1| \ge |\lambda_2| \ge \cdots \ge |\lambda_n|$. Write an octave/Matlab function, which determines the largest eigenvalue with respect to absolute value and the corresponding eigenvector using the *power iteration*:
 - (0) Pick a nonzero $\mathbf{v} \in \mathbb{R}^n$.
 - (1) Evaluate $\mathbf{w} = A\mathbf{v}$.
 - (2) Evaluate $\mathbf{v} = \mathbf{w}/\|\mathbf{w}\|$ and repeat step (1).

Iteration is stopped once \mathbf{v} is a good enough approximation to an eigenvector. The corresponding eigenvalue is then $\lambda = \mathbf{v}^T A \mathbf{v}$. (Why?)

Assume now that $A \in \mathbb{R}^{n \times n}$ is a symmetric matrix with above properties. Adapt the power iteration to the 'simultaneous power iteration' (the QR iteration) like this:

- (0) Pick linearly independent $\mathbf{v}_1, \dots, \mathbf{v}_m \in \mathbb{R}^n$, $m \le n$, and assemble them into a matrix $V = [\mathbf{v}_1, \dots, \mathbf{v}_m]$.
- (0') Determine the QR decomposition of V; V = QR
- (1) Evaluate W = AQ.
- (2) Determine the QR decomposition of W and repeat step (1).

The iteration is stopped once the columns of $Q = [\mathbf{q}_1, ..., \mathbf{q}_m]$ are good enough approximations to eigenvectors of A. The corresponding eigenvalues are again obtained from $\lambda_k = \mathbf{q}_k^T A \mathbf{q}_k$. Why did we add the requirement that A is a symmetric matrix?

Test both methods and compare them with the built-in ones on some (not too big) test cases.