Assigment 1

Solve the following five exercises. Each exercise is worth three points. Solutions must be submitted by 24.3.2024. Use the link on e-ucilnica to turn in your work. The submission must be in pdf format.

Exercise 1

a) Find the tight asymptotic bound of the algorithm given below.

Algorithm 1

```
1: function FUN(int n, int m)
2:     while n > 0 do
3:         m++;
4:         for (int i = m; i > 1; i = i/4) do
5:             UpdateData(n, i) //θ(n)
6:         end for
7:         m– –
8:         for (int i = 3m; i > m ; i = i/2) do
9:             UpdateDataFast(n, 3m − i) //θ(log n)
10:        end for
11:        n– –
12:    end while
13:    return 0
14: end function
```

b) Find the tight asymptotic bound of the following function f(a,s,e) in relation to e-s. Hint: a - array, s - start, e - end

```
int f(int[] a, int s, int e) {
   if (s == e)
       return 7;
   int p1 = 0;
   int j = e - s;

   for(int i = j; i > 1; i-- )
       p1 += a[s + i] * a[e - i]

   int p2 = f(a, s, s + floor(j / 6));
   int p3 = f(a, s + floor(j / 6) + 1, e);
   return (p1+p2+p3);
}
```

c) Find the tight asymptotic bound of the following program in relation to variable N.

```
Move(N, source, dest, via){
  if(N == 0) then{
    MakeMove(N, source, dest) //O(1)
  }else{
    Move(N-1, source, via, dest);
    MakeMove(N, source, dest) //O(1)
    Move(N-1, via, dest, source);
  }
}
```

Exercise 2

Solve the following two recurrences using the Masters method.

a)

$$T(n) = 4 \cdot T(\frac{n}{9}) + \sqrt[3]{n} \tag{1}$$

b)

$$T(n) = 5 \cdot T(\frac{n}{6}) + log(n) \tag{2}$$

Exercise 3

Solve the following two recurrences using the Akra-Bazi method.

a)

$$T(n) = 2T\big(\frac{n}{4}\big) + 4T\big(\frac{n}{8}\big) + \theta(n \log n) \tag{3}$$

b)

$$T(n) = T\big(\frac{n}{9}\big) + T\big(\frac{n}{4}\big) + T\big(\frac{n}{36}\big) + \sqrt{n^3} \tag{4}$$

Exercise 4

a) Estimate the upper asymptotic bound of the given recurrence using the recursion-tree method.

$$T(n) = 8T(\Big\lfloor \frac{n}{2} \Big\rfloor) + 27T(\Big\lfloor \frac{n}{3} \Big\rfloor) + n^3 \tag{5}$$

b) Prove the result you found in a) using the substitution method. (2 points)
d) Find the tight asymptotic bound of the given recurrence. (With any method)

Exercise 5

a) Solve the following recurrence using the annihilator method. Find the exact solution. (2 points)

$$T(n) = T(n-1) + n^2 + n - 1; T(0) = 1 \tag{6}$$

b) Solve the following recurrence using the annihilator method. Find the exact solution. (2 points)

$$\begin{aligned} T(n) = 9T(n-1) - 28T(n-2) + 36T(n-3) - 16T(n-4) \\ T(0) = 2; T(1) = 8 \\ T(2) = 28; T(3) = 112 \end{aligned}$$