

1. Parametrize implicitly given curves below:

- (a) $x^2 - y = 0$
- (b) $x = f(y)$, where $f: \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function,
- (c) $(x - p)^2 + (y - q)^2 = r^2$ for fixed $p, q, r \in \mathbb{R}$,
- (d) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

2. Parametrizations

$$\mathbf{p}_1(t) = (\cos t, \sin t, t)$$

$$\text{in } \mathbf{p}_2(t) = (\sin t, \cos t, t)$$

determine two *helices* in \mathbb{R}^3 .

Find the points of intersection of these two curves. What is the angle of intersection at these points?

3. Find a parametrization of the epicycloid (and the hypocycloid), i.e. the curve formed by tracing a point on the circumference of a circle with radius r which rolls around (inside) a fixed circle of radius R .

4. **Intersection points of planar polygonal chains.** Write an octave function $P = \text{presecisce}(A, B)$ that determines whether two line segments $\overline{A_1A_2}$ and $\overline{B_1B_2}$ intersect and if so returns the intersection. The line segments are represented by matrices

$$A = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix} \text{ in } B = \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix}.$$

Two polygonal chains K and L are defined by the sequence of line segments $\overline{A_1A_2}, \overline{A_2A_3}, \dots$ and $\overline{B_1B_2}, \overline{B_2B_3}, \dots$. Write a function $P = \text{presecisca}(A, B)$ that returns all the intersections of the polygonal chains K and L . The points that define the curves K and L and the return value P are represented by matrices:

$$A = \begin{bmatrix} x_1 & x_2 & \cdots & x_k \\ y_1 & y_2 & \cdots & y_k \end{bmatrix}, B = \begin{bmatrix} u_1 & u_2 & \cdots & u_\ell \\ v_1 & v_2 & \cdots & v_\ell \end{bmatrix} \text{ and } P = \begin{bmatrix} s_1 & s_2 & \cdots & s_m \\ t_1 & t_2 & \cdots & t_m \end{bmatrix}.$$



5. **Intersection points of two parametrically given curves.** We are given parametrizations of two curves, K and L , in the plane \mathbb{R}^2 . Our task is to find all the points of intersection of K and L .

Let $\mathbf{p}(t)$ and $\mathbf{q}(t)$ be the parametrizations of K and L defined on intervals $I = [a, b]$ and $J = [c, d]$ respectively.

Find the points of intersection using the following procedure:

- (a) Divide the intervals I and J into subintervals of length $h > 0$, where h is sufficiently small.

- (b) Approximate K and L with polygonal chains determined by evaluating the parametrizations at subdivision points of I and J and find the intersections of the polygonal chains.
- (c) Use the points of intersection of the polygonal approximations as an initial guess for Newton's iteration, which will determine the actual intersection points of K and L (much) more accurately.

You will also need the derivatives of the parametrisations $\dot{\mathbf{p}}$ and $\dot{\mathbf{q}}$ in order to use Newton's iteration. Write the function `P = presekKrivulj(p, pdot, intp, q, qdot, intq, h)` which returns the intersections of two parametrized curves.