

# 1 Approximation algorithms

## Multiprocessor scheduling

Given  $n$  jobs, each with its length  $c_i$  find the minimum time (makespan) to schedule all jobs on  $m$  processors.

1. Estimate optimal solution. (two estimations)
2. Try to solve the problem with greedy method for 2 processor. In this case you take job  $i$  and schedule it to the currently most free processor. Show that this approach is a 1.5 approximation.
3. Generalize last exercise for  $m$  processors. What is the generalized approximation factor?
4. You sort the jobs in decreasing order by their length and then use the same algorithm as before starting with longest job first. What is the approximation factor in this case? Hint: Consider job  $m + 1$ .

## Bin filling

You are given  $n$  objects. Size of object  $i$  is  $s_i \in [0, B)$ . You have to put all the objects into minimal number of bins, where each bin is of size  $B$ .

1. Use Next Fit algorithm. This algorithm takes object  $i$  and puts it into bin  $j$  if it has enough space. If it does not have enough space it considers bin  $j + 1$ . When the algorithm takes new job it tries to fit it into the last bin it visited before. Show that this algorithm is 2-approximation.
2. Use First Fit algorithm. Similar algorithm but it puts object  $i$  in the first bin that has enough space to accommodate it. Prove that this algorithm is also 2-approximation. What is the worst case you can construct? Hint: This is actually an 1.7 - approximation algorithm.