1 Probabilistic analysis

Use indicator random variable to solve the following problems.

1.1 Sum of n dice

What is the expected value of the sum of n six sided dice throws.

1.2 Hat-check problem

Each of n customers gives a hat to a hat-check person at a restaurant. The hat-check person gives the hats back to the customers in a random order. What is the expected number of customers who get back their own hat?

1.3 Inversions

Given an array of n randomly permutated distinct numbers marked A_i for $i = 1 \dots n$. If $i < j$ and $A_i > A_j$, then the pair (i, j) is called an inversion. Use indicator random variables to compute the expected number of inversions.

1.4 Balls and bins

You are given n bins and n balls. You throw each ball into the bins. Assume each ball can equally likely fall into any bin.

a) What is the expected number of empty bins?
b) What about bins with exactly one ball?
c) What about bins with exactly two balls?
d) What about bins with more than 1 ball?

1.5 Hire assistant

As in the case of hire assistant you heard of in lectures. What is the probability that the algorithm hires.

a) Exactly once.
b) Exactly n times.
c) Exactly twice.

1.6 Random generator

You are given a function BiasedRandom() which returns TRUE with probability p and false with probability $1 - p$. $0 < p < 1$.

a) Construct a function UnbiasedRandom() that uses BiasedRandom() which must return TRUE with probability $\frac{1}{2}$ and FALSE with probability $\frac{1}{2}$.
b) Prove that your function works.
c) Analyse time complexity of UnbiasedRandom().